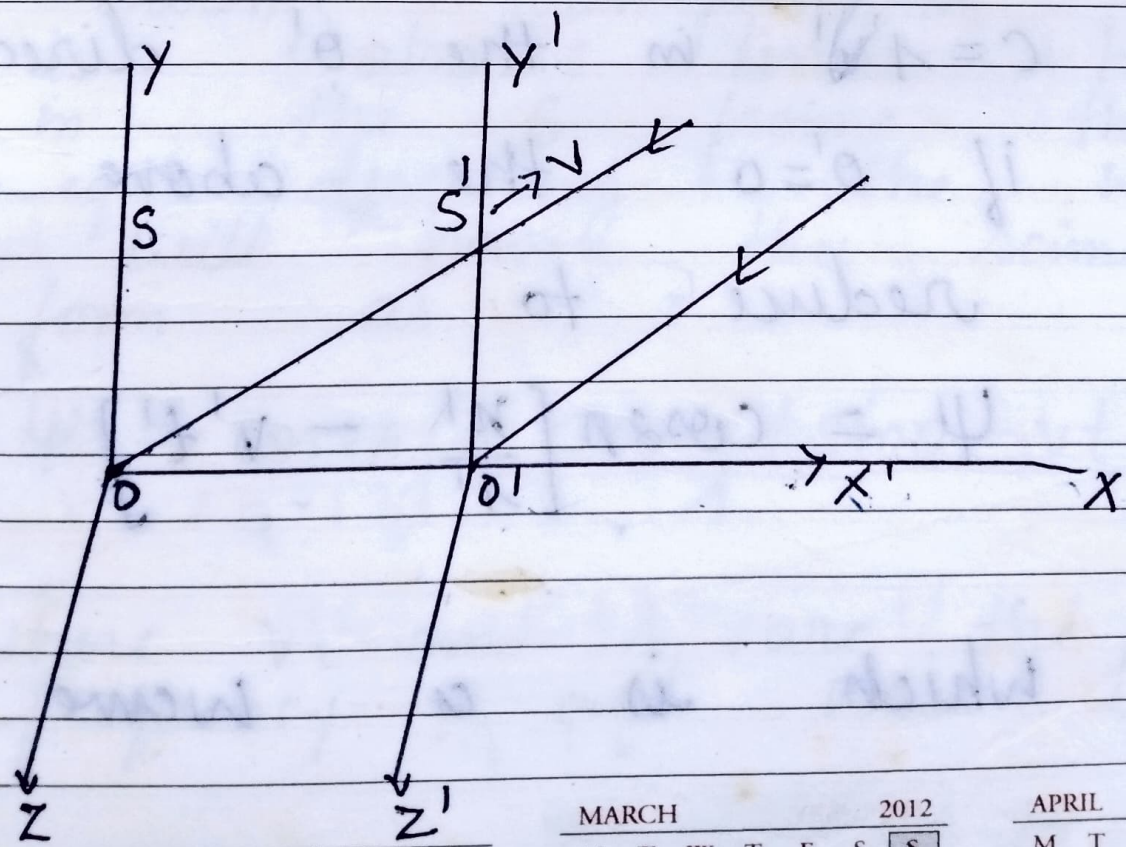


Aberration of Light: -

Let us consider a plane monochromatic wave of unit amplitude emitted from a source at the origin of a frame S' moving in x direction with a uniform velocity with respect to a fixed frame S . Let the ray or wave normals be in (x', y') plane making an angle θ' with the x' -axis.



The wave eqⁿ of such a wave is given by

$$\psi = \cos 2\pi \left[\frac{n' \cos \theta' + \gamma' \sin \theta' - v' t'}{\lambda'} \right] \quad \text{--- (i)}$$

where λ' and v' are the wave length and frequency respectively of light emitted in S' frame.

The above eqⁿ is a simple periodic function of unit amplitude moving with a velocity.

$c = \lambda' \nu'$ in the θ' direction

if $\theta' = 0$ the above eqⁿ reduce to

$$\psi = \cos 2\pi \left[\frac{n'}{\lambda'} - v' t' \right] \quad \text{--- (ii)}$$

which is a wave

moving in position of x' -axis direction

if $\theta' = \pi/2$ it reduce to

$$\psi = \cos 2\pi [Y'/\lambda' - v't'] \text{ --- (iii)}$$

which is a wave moving in position x' direction.

In the S frame these wave fronts will be plane as a plane transform as a plane due to Lorentz transformation which is also linear. hence in the S frame the eqn describing the wave will have the same form as

$$\psi = \cos 2\pi \left(\frac{n \cos \theta + \gamma \sin \theta}{\lambda} - vt \right) \text{ --- (iv)}$$

where ν and λ are the frequency and wave length

in the S. frame and
 the velocity $c = \lambda \nu$ and θ
 is the angle that the
 ray makes with n -
 axis.

using Lorentz' transforma-
 tion eqⁿ we have

$$n' = \frac{n - vt}{\sqrt{1 - \beta^2}}$$

and $t' = \frac{t - vn/c^2}{\sqrt{1 - \beta^2}}$

where $\beta = v/c$

∴ from eqⁿ (1) we have

$$\psi = \cos 2\pi \int \frac{1}{\lambda'} \cdot \frac{n - vt}{\sqrt{1 - \beta^2}} = \cos \theta' + \gamma \frac{\sin \theta}{\lambda'}$$

$$- \nu' \left(t - \frac{vn}{c^2} \right) \frac{1}{\sqrt{1 - \beta^2}}$$

(v)

Wednesday 29

on
werearranging
have

the terms

$$\psi = \cos 2\pi \left[\frac{(\cos \theta' + \beta)n}{\lambda' \sqrt{1-\beta^2}} + y \frac{\sin \theta'}{\lambda} \right.$$

$$\left. - \frac{(\beta \cos \theta' + 1) v' t}{\sqrt{1-\beta^2}} \right] \quad \text{--- (vi)}$$

which is the form of the wave in the S frame and must be identical with eqn (4) equating the co-efficients of n, y, z in eqn (4) and of

we get

$$\frac{\cos \theta}{\lambda} = \frac{\cos \theta' + \beta}{\lambda' \sqrt{1-\beta^2}} \quad \text{--- (vii)}$$

$$\frac{\sin \theta}{\lambda} = \frac{\sin \theta'}{\lambda'} \quad \text{--- (viii)}$$

$$\text{and } v = \frac{v' (1 + \beta \cos \theta')}{\sqrt{1 - \beta^2}} \quad \text{--- (ix)}$$

and the velocity of propagation of light remains the same as

$$c = v = v'$$

Dividing eqn (viii) by (vii)

we have

$$\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{\sin \theta' \sqrt{1 - \beta^2}}{\cos \theta' + \beta}$$

This is the relativistic eqn for the observation of light.